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A Spectral Iterative Technique with Gram–Schmidt Orthogonalization

PETER M. VAN DEN BERG AND WALTER J. GHIJSEN

Abstract — **Iterative schemes based on the minimization of the integrated square error are discussed. In each iteration a basis function is generated in such a way that it is linearly related to the residual error of the previous iteration. A complete orthogonalization of all of these basis functions leads to an optimal convergent scheme for some choices of the basis functions. In order to reduce the computer storage needed to store all of the basis functions, we present an incomplete orthogonalization scheme that still yields an efficient computational method. In this scheme a limited number of basis functions has to be stored. Some numerical results with respect to some representative field problems illustrate the performance of the various versions of the iterative schemes suggested here.**

I. Introduction

The spectral iterative technique (SIT), developed by Bojarski [1] and Ko and Mittra [2], has been applied to a wide class of radiation and scattering problems. Convergence problems arising in the spectral iterative technique, which are serious at times, have been eliminated by van den Berg [3] by minimizing the integrated square error in the boundary conditions on the pertinent radiating or scattering object. The convergence has substantially been improved by using all available functions of the previous iteration in the minimization procedure of each iteration (CST3-scheme [4]). In each iteration of the iterative schemes a basis function is generated in such a way that it is linearly related to the residual error of the previous iteration. MacKay and McCowen [5] have suggested a full orthogonalization of all basis functions in order to improve the convergence in an optimum way. This requires all basis functions to be stored in the computer; hence sufficient computer memory must be available. The latter authors therefore suggest that a complete orthogonalization may not be necessary and that the number of basis functions to be orthogonalized can be limited to a small number. However, in this case, the convergence can decrease dramatically after a number of iterations.

In the present paper we discuss an incomplete orthogonalization scheme where we take into account a limited number of basis functions generated in the last few iterations; however, in contrast to [5], we also use the appropriate estimate of one of the previous iterations as a function to which all the relevant basis functions have to be orthogonalized. This maintains the speed of convergence. Further, in one of the most simple forms, the latter scheme turns out to be equivalent to the contrast-source-truncation technique CST3 [4]. The latter is a truly iterative technique, because it needs the functions of the previous iteration only.

II. The Operator Equation

We consider a field computation problem in terms of an integral equation of the form [4]

$$\int_D K(x-x')f(x')\,dx' = g(x), \qquad \text{when } x \in D \tag{1}$$

where D is the domain of observation. Then, (1) is equivalent to

$$Kf = g, \qquad \text{when } x \in D. \tag{2}$$

Further, we introduce the inner product of two functions f and g as (the bar denotes a complex conjugate)

$$\langle f, g\rangle = \int_D \overline{f(x)}g(x)\,dx \tag{3}$$

while the norm of a function f is defined as $\|f\| = \langle f, f\rangle^{1/2}$. We further introduce the characteristic function $\chi_D(x) = 1$ when $x \in D$, and $\chi_D(x) = 0$ when $x \in D'$, where D' is the subdomain outside the domain D of observation.

Introducing the spatial Fourier transform of a function f as $\tilde{f} = F\{f\}$, the Fourier transform of the operator expression Kf of (2) can be written as the product of the Fourier transforms $\tilde{K} = F\{K(x)\}$ and $F\{\chi_D f\}$; thus the operator expression can be written as

$$Kf = F^{-1}\{\tilde{K}F\{\chi_D f\}\}. \tag{4}$$

III. Iterative Approximation with Gram–Schmidt Orthogonalization

In our iterative approximation we construct a sequence of functions $\{f_n,\ n = 0,1,2,3,\cdots\}$ such that the norm of the residual in the operator eq. (2),

$$ERR_n = \langle r_n, r_n\rangle^{1/2}, \qquad \text{with } r_n = Kf_n - g \tag{5}$$

decreases with increasing n in an optimum way. The procedure starts with an initial guess f_0 with the associated residual r_0. At each step of the iterative procedure, we write

$$f_n = f_{n-1} + \alpha_n f_n^c, \qquad n = 1,2,3,\cdots \tag{6}$$

where, in each step, f_n^c is a correction function and where the complex parameter α_n is chosen such that the error ERR_n is

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P. M. van den Berg is with the Laboratory of Electromagnetic Research, Department of Electrical Engineering, Delft University of Technology, P.O Box 5031, 2600 GA Delft, The Netherlands.
W. J. Ghijsen was with the Department of Electrical Engineering, Delft University of Technology, Delft, The Netherlands. he is now with Phonon, Inc., Simsbury CT 06070.
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minimal. This results in a new residual:

$$r_n = r_{n-1} + \alpha_n K f_n^c \tag{7}$$

and the value of α_n leading to minimal error

$$\alpha_n = -\langle K f_n^c, r_{n-1}\rangle / \|K f_n^c\|^2. \tag{8}$$

In our iterative scheme, we generate in each iteration a basis function ϕ_n in such a way that ϕ_n is linearly related to the residual of the previous iteration, say $\phi_n = L r_{n-1}$, in which L is a linear operator. For our relevant operator expression we now take (according to the contrast-source-truncation techniques [3], [4])

$$\phi_n = L r_{n-1} = F^{-1}\left\{ [\tilde{K}]^{-1} F\{\chi_D r_{n-1}\}\right\}. \tag{9}$$

This choice has been inspired by the SIT, suggested by Bojarski [1] and Ko and Mittra [2], to solve the operator eq. (2) directly without any error minimization.

One possible way is to take $f_n^c = \phi_n$. Then, we observe that in the nth iteration the approximate estimate f_n can be written as a linear combination of f_0 and $f_m^c = \phi_m$ $(m = 1,2,\cdots,n)$ with expansion coefficients α_m,

$$f_n = f_0 + \sum_{m=1}^{n} \alpha_m f_m^c. \tag{10}$$

However, an optimum solution for the chosen values of f_0 and f_m^c $(m = 1,2,\cdots,n)$ is arrived at by linearly combining these functions with a number of unknown expansion coefficients as

$$f_n = \alpha_{n0} f_0 + \sum_{m=1}^{n} \alpha_{nm} f_m^c \tag{11}$$

and minimizing the integrated square error. Then, we end up with a linear system of $n+1$ algebraic equations for these expansion coefficients α_{nm}, $(m = 0,1,\cdots,n)$ [4].

A. Complete Orthogonalization

For a given set of basis functions, optimal minimization of the integrated square error in the nth iteration (leading to a linear system of $n+1$ algebraic equations) is equivalent to a procedure requiring that (6)–(8) apply while the following orthogonality relations hold: $\langle K f_n^c, K f_0\rangle = 0$, when $n \neq 0$, and $\langle K f_n^c, K f_m^c\rangle = 0$, when $n \neq m$. These orthogonalization requirements can be met by constructing the correction functions f_n^c from the variational functions ϕ_n as follows:

$$f_n^c = \phi_n - \beta_{n0} f_0 - \sum_{m=1}^{n-1} \beta_{nm} f_m^c, \qquad n = 1,2,3,\cdots \tag{12}$$

where the coefficients β_{n0} and β_{nm} can be found from a Gram–Schmidt orthogonalization procedure. This scheme is similar to that of MacKay and McCowen [5]; the only difference is that we also require the orthogonality relation with respect to the initial estimate. For a zero initial estimate both schemes are identical. The disadvantage of this method is that for an increasing number n of iterations we have to store the functions $\{K f_m, m = 1,2,\cdots,n\}$ of all previous iterations.

B. Incomplete Orthogonalization

A less optimal procedure, but one involving only a limited number of correction functions, can be developed by taking the function $K f_n^c$ only orthogonal to a limited number of N functions $K f_m^c$ which were obtained from only the N previous iterations and which are orthogonal to the function $K f_q$ arrived at in the iteration with ordinal number $q = n - N - 1$. This scheme is identical to that of (12) in the first N iterations and then continues as

$$f_n^c = \phi_n - \beta_{nq} f_q - \sum_{m=q+1}^{n-1} \beta_{nm} f_m^c, \qquad n = N+1, N+2, N+3, \cdots$$
$$q = n - N - 1 \tag{13}$$

where the coefficients β_{nq} and β_{nm} can be found from a Gram–Schmidt orthogonalization procedure. Equations (6)–(9) and (12), for $n = 1,2,\cdots,N$, and (13), for $n = N+1, N+2, N+3,\cdots$, constitute our iterative procedure for a limited number of N stored functions $K f_m^c$. This scheme differs from the reduced scheme suggested by MacKay and McCowen [5] in the requirement of the orthogonality with respect to f_q. Omitting the second term in (13) yields the reduced scheme of MacKay and McCowen. However, this can influence the speed of convergence negatively. How the convergence varies as a function of N will be shown for two specific examples in the next sections.

C. The CST3 Scheme

When we take N equal to 1, (13) can be written as

$$f_n^c = \phi_n - \beta_{n,n-2} f_{n-2} - \beta_{n,n-1} f_{n-1}^c, \qquad n = 2,3,4,\cdots. \tag{14}$$

Using $f_{n-2} = f_{n-1} - \alpha_{n-1} f_{n-1}^c$, which follows from (6), we obtain

$$f_n^c = \phi_n - \beta_{n,n-2} f_{n-1} - \left(\beta_{n,n-1} - \alpha_{n-1}\beta_{n,n-2}\right) f_{n-1}^c \tag{15}$$

and we end up with a scheme that is identical to the CST3 scheme of van den Berg [4]. Note that this is a truly iterative scheme, because only the functions of the previous iteration are taken into account. This scheme is originally derived by improving van den Berg's CST and CCST scheme [3].

IV. Scattering by a Strip

One of the canonical problems in electromagnetics and acoustics is the two-dimensional scattering of a time-harmonic (time factor $\exp(-i\omega t)$) plane wave by a strip (Fig. 1). In electromagnetics, we consider the scattering of a TM-polarized wave (the electric field vector is parallel to the edges of the strip). This problem is identical to that of acoustic scattering by a soft strip. For this problem, Ko and Mittra [2] have presented their spectral iterative technique (SIT). We apply the iterative schemes of the former section to solve the one-dimensional operator equation of this simple example. We take the zero initial guess. To perform the one-dimensional Fourier transforms numerically we use a fast fourier transform (FFT) routine. All inner products over the domain D $(-a < x < a)$ of the strip are calculated numerically by a simple summation of the function values at the sample points.

After some normalization we have in the operator equation the known function $g(x) = 1$, $x \in D$, and the Fourier transform of the kernel $\tilde{K} = (k^2 - \alpha^2)^{-1/2}$. In order to cope with this branch point in the numerical inverse Fourier transform, we introduce slight losses in the embedding in accordance with the condition of causality by taking a complex wavenumber $k = (2\pi/\lambda)(1 + 0.01i)$, where λ is the wavelength.

In the numerical computations we use a 4096-point FFT routine. The number of sample points on the strip (of width $2a$) amounts to 81 and 35 when $ka = 5$ and 1, respectively. In Figs. 2 and 3, we present the numerical values of the normalized error $ERR_n/\|g\|$ for $ka = 5$ and $ka = 1$. The solid lines represent the results of the incomplete orthogonalization scheme for different values of N, where N is the number of correction functions taken

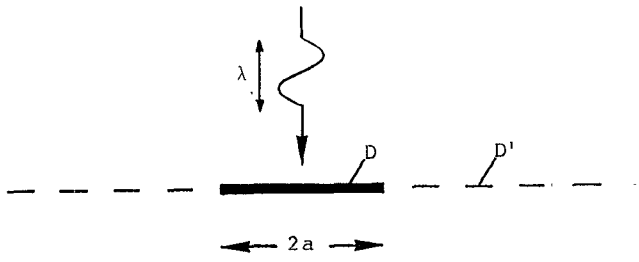

Fig. 1. Plane-wave scattering by a strip.

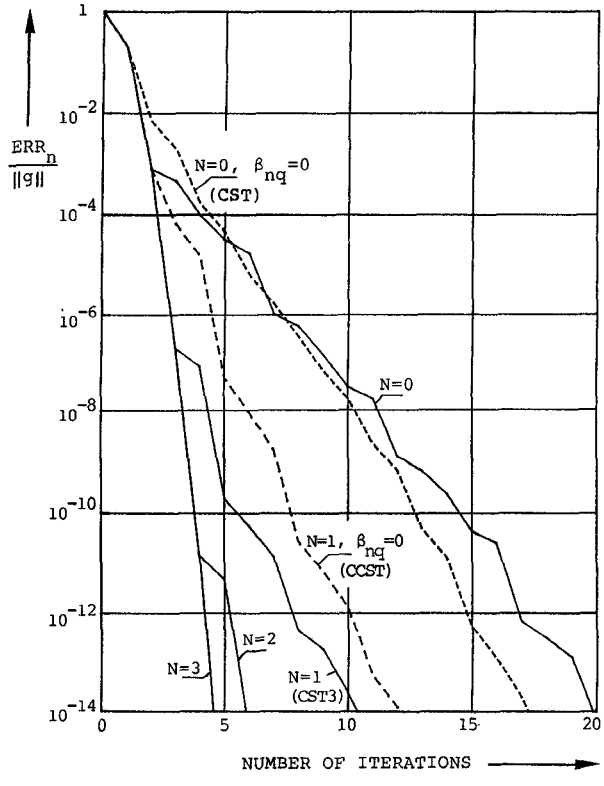

Fig. 2. The normalized error ERR_n as a function of the number of iterations for the scattering problem of a strip ($ka = 5$).

into account. The scheme for $N=1$ is the CST3 scheme [4]. The broken lines represent the results of the incomplete orthogonalization scheme if we enforce the value of β_{nq} of the second term of (13) to zero. In this latter case, the two schemes for $N=0$ and $N=1$ coincide with the so-called CST and CCST schemes [3], respectively.

V. The Electric Field Problem of an Interdigital Transducer

The analysis of the computation of the electric field excited by an interdigital transducer in a multilayered structure [6] can be reduced to the solution of a boundary integral equation of the type given in (1). As a test configuration to demonstrate the performance of the different schemes we consider the electrostatic field problem of a periodic configuration of two electrodes having a prescribed potential of opposite polarity in vacuum [7]. In this periodic case, we have to use the periodic Fourier transform. We again apply the iterative schemes to solve the one-dimensional operator equation of this example. To perform the one-dimensional Fourier transforms numerically, we use a 1024-point FFT routine. All inner products over the domain $D(L/2 < x < 3L/2$ and $5L/2 < x < 7L/2)$ of the two electrodes

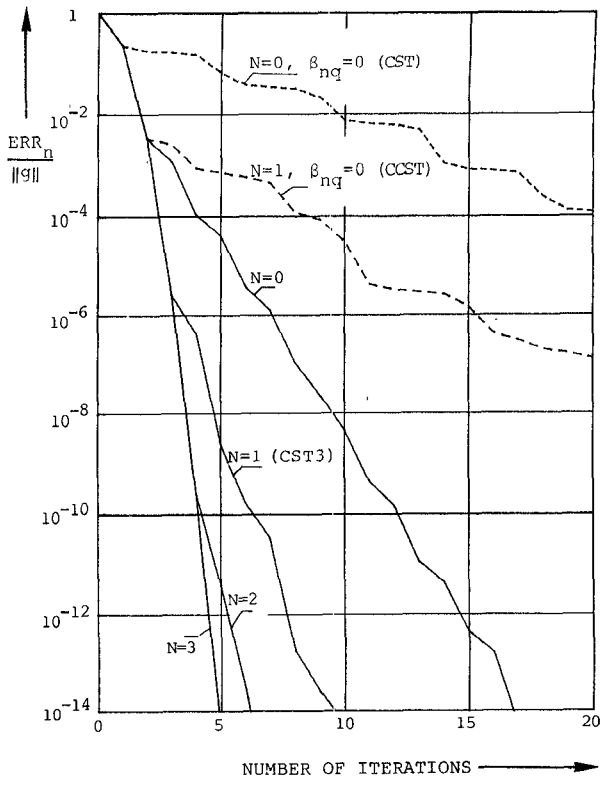

Fig. 3. The normalized error ERR_n as a function of the number of iterations for the scattering problem of a strip ($ka = 1$).

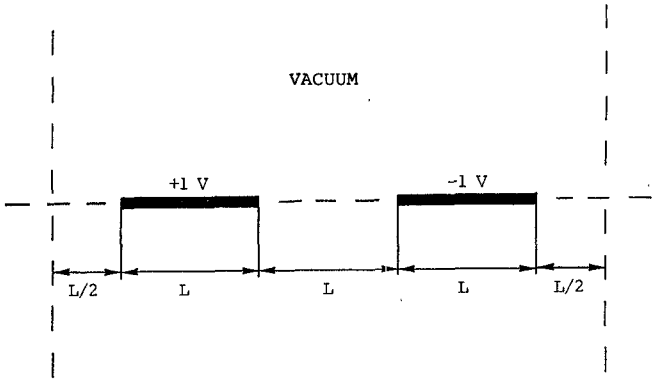

Fig. 4. Interdigital transducer of periodic strip electrodes

of width L in a spatial period of the configuration (Fig. 4) are calculated numerically by a simple summation of the function values at the sample points. The number of sample points on each electrode amounts to 256.

After some normalization we have in the operator equation the known function $g(x)=1$ when $L/2 < x < 3L/2$, $g(x)=-1$ when $5L/2 < x < 7L/2$, and the Fourier transform of the kernel $\tilde{K} = |\alpha|^{-1}$, $\alpha = 2\pi n/4L$ $(n = 0, \pm 1, \pm 2, \cdots)$. In order to cope with the singularity $\alpha = 0$ in the numerical inverse Fourier transform, we put $\tilde{K}(\alpha) = 0$, for $\alpha = 0$. Subsequently, the average value of the potential function $K\phi_n$ obtained over the domain D of the electrodes is enforced to zero (in view of the asymmetric excitation). For a detailed description of the generation of the basis functions ϕ_n and corresponding potential $K\phi_n$, we refer the reader to [7, table II].

In Fig. 5, we present the numerical values of the normalized error $ERR_n/\|g\|$. The solid lines represent the results of the incomplete orthogonalization scheme for different values of N,

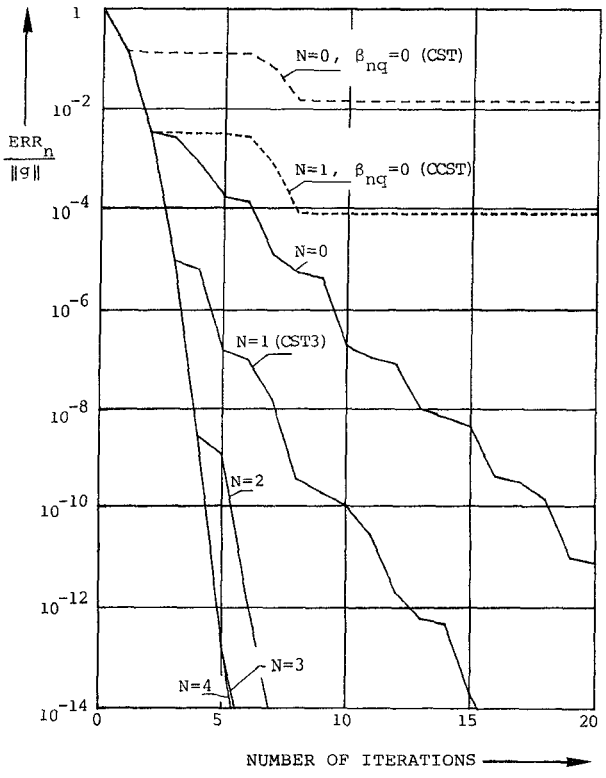

Fig. 5. The normalized error ERR_n as a function of the number of iterations for the electric field problem of an interdigital transducer.

where N is the number of correction functions taken into account. The scheme for $N=1$ is the CST3 scheme [4], [6]. The broken lines represent the results of the incomplete orthogonalization scheme if we enforce the value of β_{nq} of the second term of (13) to zero. In this latter case, the two schemes for $N=0$ and $N=1$ coincide with the so-called CST and CCST schemes [3], [7], respectively.

To conclude we observe that a full orthogonalization of all basis functions is in many cases no longer necessary when the incomplete orthogonalization scheme is followed. It appears that the CST3 scheme is a very efficient computational scheme, in view of achieving the right balance between computer memory and computer time. Another approach is one in which the available computer determines the number of correction functions that will be taken into account in our incomplete orthogonalization scheme. The available computer memory for the user is the upper limit of this number of correction functions.

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High-Speed GaAs Dynamic Frequency Divider Using a Double-Loop Structure and Differential Amplifiers

MASAFUMI SHIGAKI, MEMBER, IEEE, TAMIO SAITO, HIROTSUGU KUSAKAWA, AND HIROSHI KURIHARA

Abstract—**New GaAs 2.0–8.0 GHz and 6.0–10.5 GHz dynamic frequency dividers have been developed. These dynamic dividers have a double-loop structure using a differential amplifier for high-speed and stable operation despite supply voltage fluctuations. This structure operates from one voltage supply. An advanced WSi self-aligned gate process technology (1.0 μm long gate) was used to improve the high-frequency characteristics of the FET.**

I. Introduction

Satellite and multiplex communication systems need oscillators of small size and high stability. High-performance phase-locked oscillators can satisfy these demands, but require the development of high-speed frequency dividers. Dynamic frequency dividers can operate at a higher frequency than static ones, because their propagation delay time is shorter [1]–[3]. However, because of the direct-coupled feedback circuit and the use of a common-source FET for the inverter, they are sensitive to supply voltage fluctuation, and cannot operate at variations of ±10 percent. Such conventional dividers require many power supplies.

This paper describes a newly developed dynamic frequency divider that overcomes these problems without sacrificing speed. This divider was constructed by connecting differential amplifiers in a double loop. This structure provides stable operation insensitive to supply voltage fluctuation, and moreover it achieves single supply operation. WSi self-aligned structure gate process technology was used to produce the IC for ease of mass production [4]. We have produced 2.0–8.8 GHz and 6.0–10.5 GHz dynamic dividers.

II. Circuit Design

The fundamental circuit configuration of the dynamic frequency divider is shown in Fig. 1. The common-source inverter was replaced by differential amplifier A. Also, the source follower buffer was replaced by differential amplifier B for more gain. A schematic of the differential amplifiers is shown in Fig. 2(a). The diode over the load resistor regulates proper drain–source voltage for high-level operation of the level-shift FET's (C, D). The self-biasing circuit was used to determine the input dc level for proper operation as shown in Fig. 2(b). A two-stage output buffer was adopted to retard reduction of the operating frequency. The first stage used a small gate width buffer for

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M. Shigaki, T Saito, and H. Kurihara are with Fujitsu Laboratories Ltd., Nakahara-ku, 211 Japan.

H Kusakawa is with Fujitsu Ltd, Nakahara-ku, 211 Japan.

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